Correcting times for different length sailing courses

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1 Introduction

The Dunsborough Bay Yacht Club (DBYC) is a small off-the-beach club on Geographe Bay, Western Australia. Club members sail mono- and multihull yachts of the following classes: Pacer, Laser, Windrush, F18 (Nacra), Flying Ant and Minnow. The fleets for club racing are usually small, (6 – 15 boats) and the courses (triangles, trapezoids, windward/return) are laid to suit the wind and it is often the case that several short races of 30-40 minutes duration are held on an afternoon of sailing. To ensure that sailors are not waiting for long periods between races, faster yachts are assigned to fleet A and sail course A, and slower yachts are assigned to fleet B and sail a shorter course B, with the aim of all yachts finishing within a 5-10-minute period.

If handicap race results are required for the combined fleet, then the *elapsed times (ET)* of yachts in fleet A sailing course A (the longer course) need to be scaled to represent elapsed times they might have had, if they had sailed the shorter course B.

We show, in the following sections:

(i) A plausible method for determining the distance a yacht sails during a race that comprises beating, reaching, and running. We call this distance the *sailing length,* and it will be longer than the *course length* which is the sum of the distances between marks rounded in the race.

(ii) A plausible method of calculating *scaled elapsed times (SET)* for each of the k yachts in fleet A they may have had if they had sailed a shorter course B

$$SET_k = ET_k \times SLR \tag{1}$$

Where SLR is the *sailing length ratio* and

$$SLR = \frac{sailing\,length\,of\,shorter\,course\,B}{sailing\,length\,of\,longer\,course\,A} \tag{2}$$

Sailing Courses A and B and Course Lengths

For the purposes of this paper, the courses A and B are both sailed around marks forming 45° right-angled triangles shown in Figure 1. The windward marks are 1 and X, the gybe marks are 2 and Y, and the leeward marks are 3 and X. Races start in the vicinity of 3 and Z and finish in the vicinity of 1 and X.

For the course A triangle, the windward leg $a = 1000$ m and the reaching legs 1-2 and 2-3 are both of length $\frac{a}{\sqrt{2}} = 707$ m (nearest metre).

For the course B triangle, the windward leg $b = 750$ m and the reaching legs X-Y and Y-Z are both of length $\frac{b}{\sqrt{2}} = 530$ m (nearest metre).

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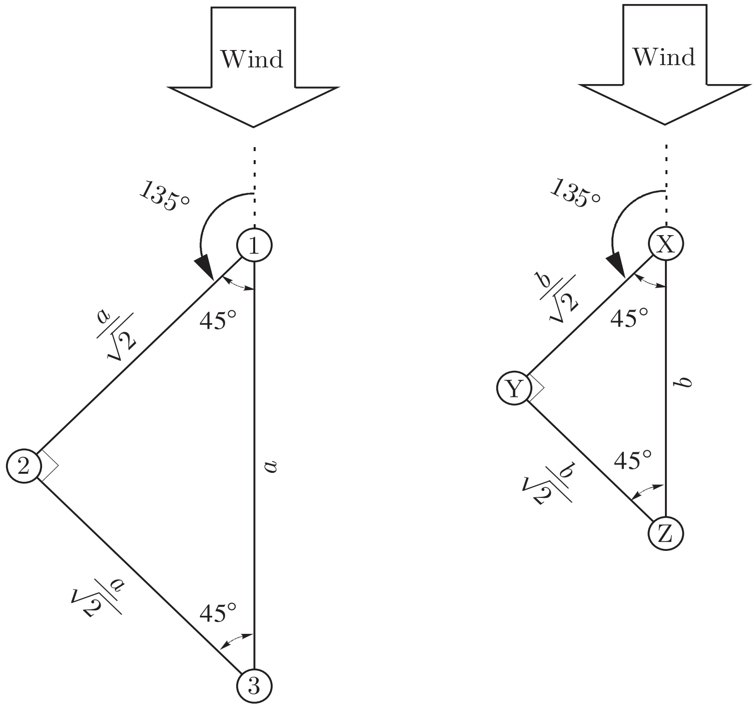

Figure 1. Triangle for course A (left), triangle for course B (right)

Course A (the longer course) in mark rounding order is: **Start–1–2–3–1–3–Finish** leaving all marks to port. Or, triangle, windward/return, beat to finish and a yacht sailing this course would have sailed 3 beats, 2 reaches and 1 run for a $course\,length = 3\left(1000\right) + 2\left(707\right) + 1\left(1000\right) = 5414$ m

Course B (the shorter course) in mark rounding order is: **Start–X–Y–Z–X–Z–Finish** leaving all marks to port. Or, triangle, windward/return, beat to finish and a yacht sailing this course would also have sailed 3 beats, 2 reaches and 1 run for a $course\,length = 3\left(750\right) + 2\left(530\right) + 1\left(750\right) = 4060$ m .

Sailing Length – Windward beat

A yacht cannot sail directly into the wind, so if she needs to sail to a windward mark the mainsail boom is set (on the port or starboard side) so that she is close hauled or beating to windward and her direction is at an angle θ to the wind, and θ (Greek symbol *theta*) is the *true wind angle*[2]. At some stage, she needs to change direction towards the mark and the mainsail boom will pass 'through the wind' and be set, close-hauled on the opposite side of the boat and her direction will again be at an angle θ to the wind. This maneuver is known as a *tack* and Figure 2 shows two possible sailing courses of a yacht on a windward beat.

Say the 1st course (dotted line) is the course 3–A–B–C–1 where she tacks 3 times, and the 2nd course (dashed line) is 3–D–1 where she tacks once. If the wind is steady and the true wind angle θ remains constant, the two sailing courses are the same length. This relationship between a course with one tack and a course with 3 tacks will hold true for courses with 2, 3, 4, 5, etc. tacks., and in general, if x is the leg length, the distance between the leeward mark 3 and the windward mark 1, then

$$sailing\,length\ (\text{upwind}) = \frac{x}{\cos\theta} = x\sec\theta \qquad \text{where } \left|\theta\right| < 90^\circ \tag{3}$$

[The notation $\left|\theta\right|$ means the magnitude of θ and if $\theta = -45^\circ$ then $\left|\theta\right| = 45$.]

[2] The true wind angle θ is measured positive clockwise and negative anticlockwise from 0° to 180° with the zero direction directly into the wind.

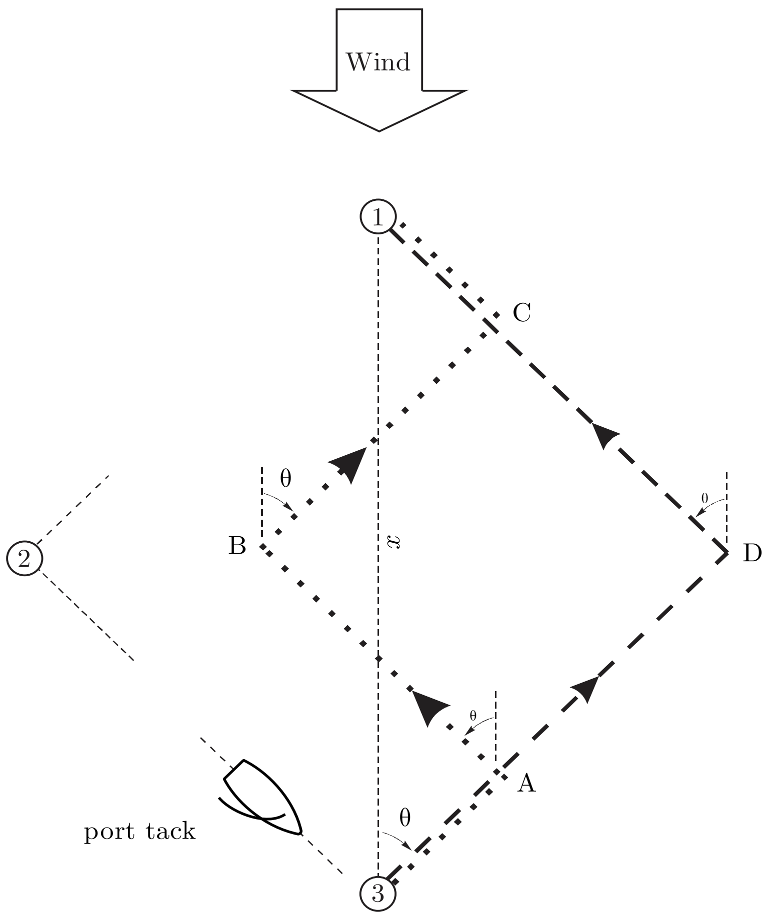

Figure 2. Sailing course options on the windward beat

Sailing Length – Downwind run

A yacht sailing directly downwind has a true wind angle $\theta = 180°$ and this will be the yacht's slowest downwind direction. To complete a downwind leg in the shortest time a yacht should sail a course with true wind angle θ of approximately 150° and an optimum value of θ for a particular yacht could be determined from its polar diagram. A yacht sailing a downwind course where $\theta = 150°$ will have its mainsail boom set to one side of the yacht or the other, and at some stage will need to change direction so that θ is once again 150° but its mainsail boom will be set on the opposite side of the yacht. At some stage during this manoeuvre, known as a *gybe,* the wind will be directly astern, and the sailor needs to avoid the boom as it swings from one side to the other.

A diagram like Figure 2 will reveal that a yacht gybing downwind on a run between a windward mark and a leeward mark that are distance x apart (x is the leg length) will sail a course with a length given by

$$\textit{sailing length}\ \text{(downwind)} = \frac{-x}{\cos\theta} = -x\sec\theta \qquad \text{where } 90° < \left|\theta\right| < 180° \tag{4}$$

[Note that equations 3 and 4 differ by a minus sign and this is because the true wind angle θ on a beat will have a magnitude less than 90° where the cosine (and secant) will be positive, and on a run, the true wind angle will have a magnitude greater than 90° and less than 180° where the cosine (and secant) will be negative.]

Sailing Length – Reach

Reaches on a yachting course usually have true wind angles with magnitudes between 110° and 150° and the fastest time for a reaching leg is achieved when the sailing direction is the same as the course direction. Hence for reaches,

$$\text{sailing length}\left(\text{reach}\right) = \text{leg length } x \tag{5}$$

Sailing Length for Courses A and B

Tables 1 and 2 show the information needed for calculating the sailing length of each leg of the course using formula (3), (4), or (5) as appropriate.

Course A: Start-1-2-3-1-3-Finish					
Leg	Point of sail	Leg length x	True wind angle θ	Sailing length formula	Sailing Length
Start-1	Beat	1000 m	45°	$SL = x\sec\theta$	1414 m
1-2	Reach	707	135°	$SL = x$	707
2-3	Reach	707	135°	$SL = x$	707
3-1	Beat	1000	45°	$SL = x\sec\theta$	1414
1-3	Run	1000	150°	$SL = -x\sec\theta$	1155
3-Finish	Beat	1000	45°	$SL = x\sec\theta$	1414
Course length		5414 m	**Course sailing length**		**6811 m**

Table 1

Course B: Start-X-Y-Z-X-Z-Finish					
Leg	Point of sail	Leg length x	True wind angle θ	Sailing length formula	Sailing Length
Start-X	Beat	750 m	45°	$SL = x\sec\theta$	1061 m
X-Y	Reach	530	135°	$SL = x$	530
Y-Z	Reach	530	135°	$SL = x$	530
Z-X	Beat	750	45°	$SL = x\sec\theta$	1061
X-Z	Run	750	150°	$SL = -x\sec\theta$	866
Z-Finish	Beat	750	45°	$SL = x\sec\theta$	1061
Course length		4060 m	**Course sailing length**		**5109 m**

Table 2

Scaled Elapsed Times

Suppose a Fleet A yacht sails Course A in an elapsed time ET of 42m 50s = 2570 s where m = minute and s = second. We could say that this yacht sailed at the rate of 2570 s/6811 m, or 377.3308 s/km where m = metre and km = kilometre.

It would be reasonable to assume that her *scaled elapsed time* (*SET)* around the shorter course B would be: $SET = 377.3308 \times 5.109 = 1927.783 \text{ s} = 32\text{m } 08\text{s}$. Since the sailing rate 377.3308 is derived from the yacht's elapsed time/sailing length of course A we could express the scaled elapsed time as

$$SET = 2570 \times \frac{5109}{6811} = ET \times \frac{sailing\,length\,course\,B}{sailing\,length\,course\,A}$$

We call the last term in the equation above the *sailing length ratio (SLR)* and

$$SLR = \frac{sailing\,length\,course\,B}{sailing\,length\,course\,A}$$

giving

$$SET = ET \times SLR$$

These last two equations are our equations (1) and (2)

We feel that the previous sections give a plausible explanation of sailing length and our equations (1) to (5) based on the assumptions:

(i) The true wind angles for a beat and run respectively are $\theta = 45°$ and $150°$,

(ii) The true wind direction and velocity remain constant during the race, and

(iii) The percentages of beat, reach and run for the two courses A and B are the same.